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LONG LINE EFFECT IN CONNECTION WITH

CAVITY RESONANCES

by

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Research Report No. PIBMRI-1148-63

for

The Air Force Office of Scientific Research
The Army Research Office
The Office of Naval Research

Grant No. AFOSR-62-295

April 17, 1963

MRI

POLYTECHNIC INSTITUTE OF BROOKLYN

MICROWAVE RESEARCH INSTITUTE

ELECTROPHYSICS DEPARTMENT

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AD 403 374

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TO BASIC DOCUMENT

AD-403 374   add report number AFOSR-4831

403 374

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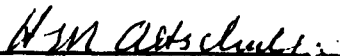
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ACKNOWLEDGMENT

ABSTRACT

When resonant cavities are separated from relatively small obstacles by long waveguides, either multiple resonances or resonant frequency shifts may occur. This long line effect is explored for both transmission and reflection cavities and is found to be much more serious in the former than in the latter case. It can usually be eliminated by matching procedures.

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I. Introduction

When cavities are separated from relatively small discontinuities by long trans-mission lines, one can, to a first order, expect both cavity resonances and resonances of the system formed by the long lines and the discontinuities adjacent to them. The latter are closely spaced and in part presumably fall into or near the cavity resonant region. Such situations can be expected to "pull" the actual cavity resonant frequencies to a larger or lesser extent, depending on the detailed parmeters involved. These "long line" effects are explored below for both transmission and reflection cavities.

Work in this connection was first stimulated by the unexpected appearance of multiple, relatively closely spaced, transmissions of a transmission cavity located be-tween two long waveguides. The subsequent difficulty of deciding on "the" cavity reson-ance and the disparity between the measured "cavity resonant frequencies" of this cavity as the measurement was repeated several times, with what seemed to be very slight vari-ations, necessitated a detailed analysis.

A simple microwave configuration which might give rise to the long line effect is shown in Fig. 1, typical patterns of transmitted power in Fig. 7 and a convenient network representation for the physical structure in Fig. 4.
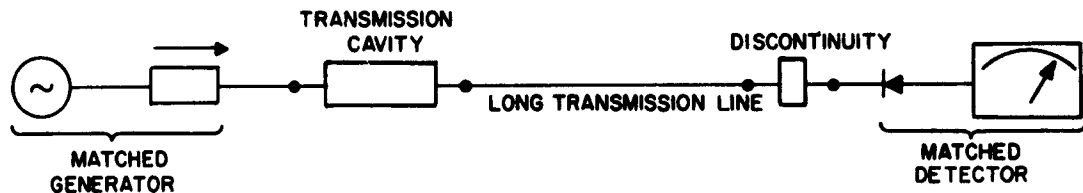
Fig. 1 Configuration which might give rise to long line effect.

It is in this representation, and a comparable one in the reflection cavity case, which is analized below.

This effect has presumably found its way into many measurements involving long waveguides and cavities, notably measurements at cryogenic temperatures, which usually necessitate transmission lines which are many wavelengths long. If not detected, elimin-ated, or understood, it may well either cause undue experimental difficulty or unnecessar-ily large errors. Fortunately, the cure, when it is practical at all, lies not in long cal-culations or elaborate calibration schemes, but in relatively simple laboratory precautions, i. e., in matching.

II. Cascaded Transmission Lines and Ideal Transformers

Since the representations of the physical structures of interest are based on ideal transformers and transmission lines, a few preliminary remarks concerning these elements

and their relation to each other are in order. The scattering transfer matrix of a two-port is known to be very useful in connection with work involving cascaded network elements. The definition of this matrix, τ, as employed here, is

$$
\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} \tau_{11} & \tau_{12} \\ \tau_{21} & \tau_{22} \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix} , \tag{1}
$$

where a and b are the complex amplitudes of the incident and reflected waves and where subscripts 1 and 2 refer to ports 1 and 2 of the two-port. It is readily seen that the scattering transfer matrix of a transmission line of length ℓ is

$$
\tau = \begin{pmatrix} e^{j\kappa\ell} & 0 \\ 0 & e^{-j\kappa\ell} \end{pmatrix} , \tag{2}
$$

that of an ideal transformer with turns ratio n:1, as shown in Fig. 2(a), can be shown to be

$$
\tau = \frac{n^2 + 1}{2n} \begin{pmatrix} 1 & N \\ N & 1 \end{pmatrix} \qquad N \equiv \frac{n^2 - 1}{n^2 + 1} \tag{3a}
$$

Fig. 2 Ideal transformer and its dual.

and that of an ideal transformer with turns ratio 1:n, as shown in Fig. 2 (b), then follows as

$$
\tau = \frac{n^2 + 1}{2n} \begin{pmatrix} 1 & -N \\ -N & 1 \end{pmatrix} . \tag{3b}
$$

Here n^2 is, of course, the insertion VSWR of the transformer and N the associated reflection coefficient magnitude i.e., $|S_{11}|$. With n restricted to the range $1 \lessgtr n \lessgtr \infty$, it follows that $0 \lessgtr N \lessgtr 1$.

The two transformers in Fig. 2 are "dual" to each other in the sense that they may be employed to represent the same structure at "dual planes", i. e., T_1 and T_2 are respectively located $\lambda_g/4$ away from T_1' and T_2'. As a consequence, a variety of representations involving transmission lines and transformers, some convenient, some not, are possible for a given lossless structure. For example, when loss is neglected, a transmission cavity can be represented to a good approximation as in Fig. 3(a), or when $n_2:1$, the dual of transformer $1:n_2$, is employed, as in Fig. 3(b). Assuming the
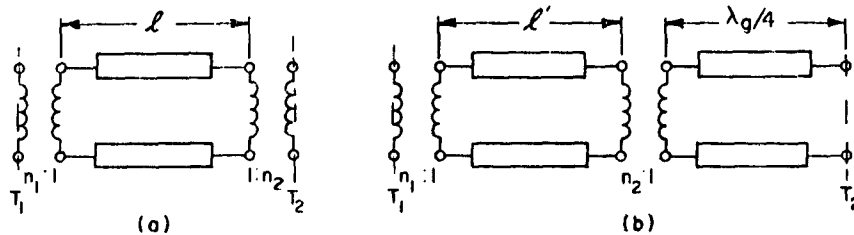

Fig. 3 Alternative representation of a transmission cavity.

transmission line length ℓ in Fig. 3(a) to be almost equal to the corresponding physical cavity length, as it usually is, the length $\ell' = (\ell + \lambda_g/4)$ in the representation of Fig. 3(b) though proper, may lead to confusion and error. Also, quite apart from physical waveguide lengths, networks with transformers disposed as in Fig. 3(a) resonate when $\ell = n\lambda_g/2$, while those with transformers disposed as in Fig. 3(b) resonate when $\ell' = (n\lambda_g/2 + \lambda_g/4)$, so that the former are seen to be more in accord with the usual notions of resonance.

Representations of the type shown in Fig. 3(b), i. e., with all transformers faced in the same direction, have been used, for example, in microwave filter synthesis[1]. For the present purpose, however, it is felt that representions such as that in Fig. 4, which use transformers in the sense of Fig. 3(a) are preferable.

III. Transmission Cavity, Long Line and Small Obstacle

Let the transmission properties of a lossless system constituted of a transmission cavity which is separated from a small discontinuity by a long transmission line be considered. The corresponding equivalent circuit is shown in Fig. 4. (Note the order of transformers: 1, 3, 2). Assuming that waveguides ℓ_1 and ℓ_2 are of the same cross-section, a single value of κ is associated with both. Since the scattering coefficient S_{21},
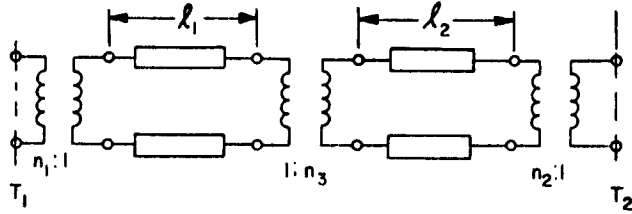
Fig. 4 Representation of transmission cavity, long line and lossless discontinuity.

which is given by

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2 = 0}$$

(4)

is of interest in connection with the transmission properties of the system, it is worth noting from eq. (1) that

$$S_{21} = \frac{1}{\tau_{11}} \, .$$

(5)

To find the system resonances (transmissions) one need therefore only examine the minima of $|\tau_{11}|$, as will be done below.

Starting with the matrices of eqs. (2) and (3), one finds on cascading that τ_{11} for the network of Fig. 4 is

$$\tau_{11} = \frac{(n_1^2 + 1)(n_2^2 + 1)(n_3^2 + 1)}{8 n_1 n_2 n_3} \left[e^{j\kappa(\ell_1 + \ell_2)} - N_1 N_3 \, e^{-j\kappa(\ell_1 - \ell_2)} - N_2 N_3 \, e^{j\kappa(\ell_1 - \ell_2)} \right. $$
$$\left. + N_1 N_2 \, e^{-j\kappa(\ell_1 + \ell_2)} \right] \, .$$

(6)

This expression is still perfectly general in that no assumptions have been made; it refers to any three ideal transformers with turns ratios in the indicated directions, separated by transmission lines of arbitrary length. On making the very good assumption that reasonably near resonance the transformers have constant values, so that only $\kappa \ell_1$ and $\kappa \ell_2$ are frequency dependent, one sees that the first factor can be discarded and only the minima of a function $F(\kappa)$, which is proportional to $|\tau_{11}|$, needs to be examined. Factoring out $\exp\left[j\kappa (\ell_1 + \ell_2) \right]$ from eq. (6), $F(\kappa)$ is taken as

$$F(\kappa) = \left| 1 - N_1 N_3 \, e^{-j2\kappa\ell_1} - N_2 N_3 \, e^{-j2\kappa\ell_2} + N_1 N_2 \, e^{-j2\kappa(\ell_1+\ell_2)} \right| . \qquad (7)$$

This function can be visualized as a planar "celestial" system as follows: It gives the variation of the distance from a point (the origin), which is fixed in space with respect to a "sun" (located at +1), to a "satellite" of a "moon" of a "planet" of the same sun.

Equation (7) is invariant to the interchange of indexes 1 and 2, so that in the case of interest it is not material whether the cavity lies to the left or the right of the long line. However, let us assume for definiteness that transformers 1 and 3 are associated with the cavity, so that N_1 and N_3 are both reasonably close to unity and ℓ_1 is short. The length ℓ_2 then represents the long line and is large, and N_2 represents the discontinuity.

The motion of the satellite is somewhat easier to picture when eq. (7) is reordered as

$$F(\kappa) = \left| 1 - N_1 N_3 \, e^{-j2\kappa\ell_1} + N_2 \, (N_1 \, e^{-j2\kappa\ell_1} - N_3) \, e^{-j2\kappa\ell_2} \right| . \qquad (8)$$

In this equation one recognizes $N_2 \left[N_1 \exp(-j2\kappa\ell_1) - N_3 \right]$ as a vector with an ℓ_1 dependent magnitude which rotates essentially at $\exp(-j2\kappa\ell_2)$, i.e., very quickly. It is at a minimum at the resonant frequency of the cavity, $2\kappa\ell_1 = 2n\pi$. When $N_2 = 0$, i.e., when there is no obstacle, eq. (8) shows $F(\kappa)$ to be the distance from an origin to a circle of radius $N_1 N_3$ centered at the point 1. The frequency associated with that point on this circle located closest to the origin is the resonant frequency of the cavity itself. However, when $N_2 \neq 0$, $F(\kappa)$ moves quickly through a series of maxima and minima. Each such minimum corresponds to a maximum value of $|S_{21}|$; the frequency associated with it is consequently taken as a system resonance point.

Figure 5 shows $F(\kappa)$ for a particular set of network parameters, though not explicitly, as the distance from the origin to the satellite path (dashed spiral). The ratio ℓ_2/ℓ_1 has been taken as 36 in this case. The cavity irises have insertion VSWR's of 19 and 39 respectively; the discontinuity has an insertion VSWR of 3. For convenience, but quite arbitrarily, the phase of $2\kappa_0\ell_2$ has been taken as $2n\pi$ at cavity resonance so that the spiral is symmetric about the line 0 - 1. In the absence of the discontinuity, $F(\kappa)$ would be the distance from the origin to the path of the planet. The circles C_1, C_2 and C_3 represent limits of sensitivity of the detection system so that transmissions corresponding to any portion of path lying within such a circle can be seen by the experimenter. For example, assuming C_1 (lowest sensitivity), cavity resonance would not be visible in the absence of the obstacle ($N_2 = 0$), but resonances become barely visable at

$$\ell_2/\ell_1 = 36$$
$$N_1 = .9$$
$$N_3 = .95$$
$$N_2 = .5$$
$$2\kappa_0\ell_2 = 2n\pi$$

C_3

C_2

C_1

CAVITY RESONANT
POINT κ_0

(SUN)
+1

ORIGIN
(FIXED POINT)

$-2\kappa'\ell_1$

PATH OF SATELLITE
(MOON IS NOT SHOWN)

$N_1 N_3$

$-2\kappa\ell_2$

PATH OF PLANET
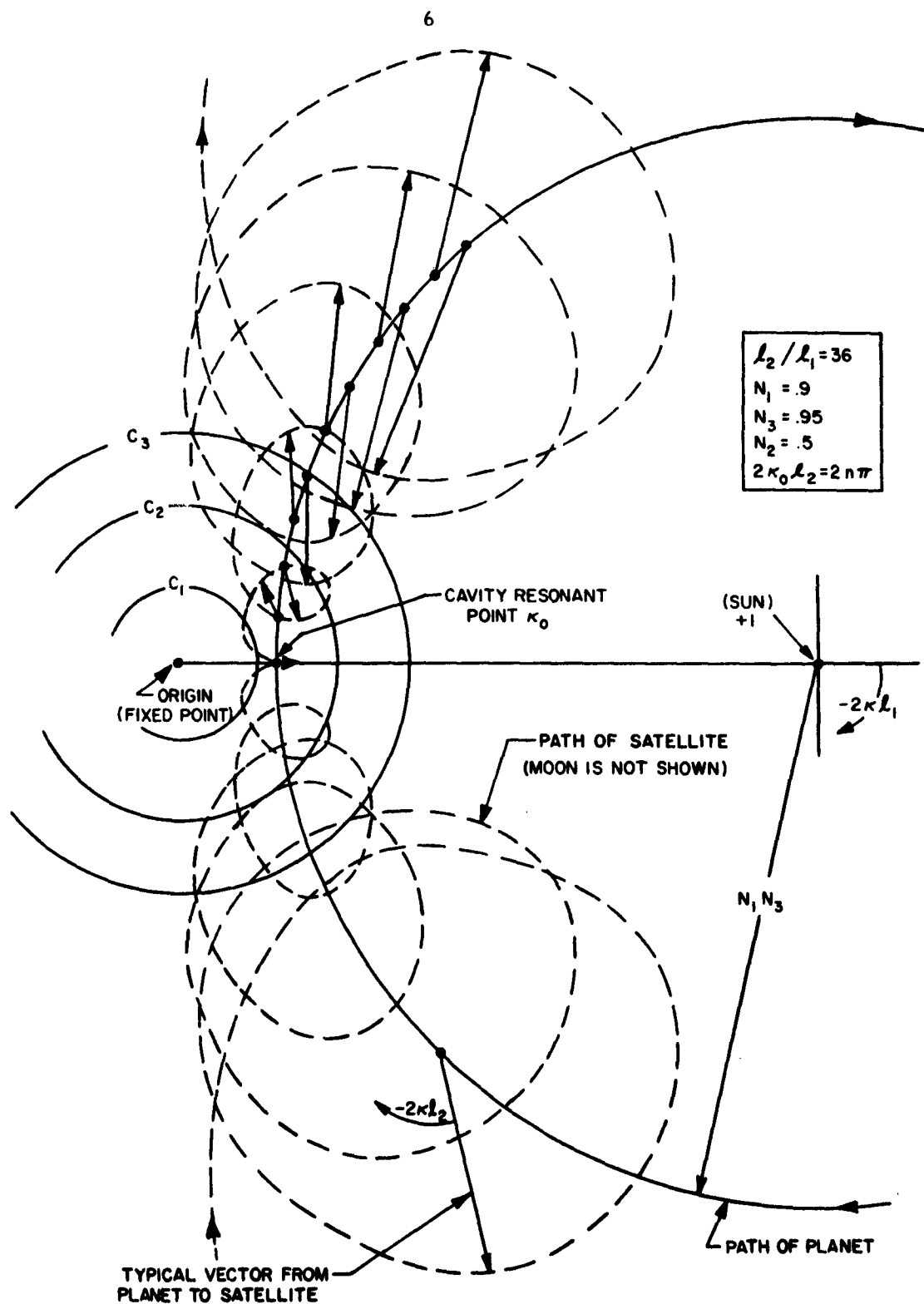
TYPICAL VECTOR FROM
PLANET TO SATELLITE

Fig. 5 Typical long line effect for a transmission cavity: $\ell_2/\ell_1 = 6$.

two different frequencies when the obstacle is present. Assuming sensitivity limits C_2 and C_3, respectively, five and seven frequency-wise roughly equispaced separate transmissions would be observable.

It can be seen that in general the cavity resonant frequency cannot be expected to coincide with one of the system transmission frequencies. The important consequence of this fact is that the cavity resonant frequency cannot be determined, except very approximately, in the presence of additional obstacles separated from the cavity by a long transmission line. When more than one obstacle is present, multiple transmissions can of course also be expected.

When the "long" line is substantially shorter than in the example of Fig. 5, the multiple resonances may no longer be observable, but a substantial difference may exist between the cavity resonant frequency and the system resonant frequency. This is demonstrated in Fig. 6 which differs from that of Fig. 5 only in that ℓ_2 / ℓ_1 now equals six and in that $2\kappa_0 \ell_2$ is no longer an even multiple of π. The figure shows clearly that the satellite moves much more slowly, as it should, that no multiple resonances lie within sensitivity circles C_1, C_2 and C_3 and that symmetry no longer exists about the 0 - 1 line. However, it is most important that the system resonant point now lies at a frequency corresponding to $1.014\kappa_0$, and since there is only a single resonant point, it may inadvertently be mistaken for the cavity resonant point.

The "long line effect" has been subjected to qualitative, though not detailed, experimental verification. The power transmitted through circuits of the type under discussion was displayed on an oscilloscope against a frequency scale. A swept source was employed. Ratios of ℓ_2 / ℓ_1 up to about 47 and a variety of discontinuities were used. Transmissions were predictably spaced and their average amplitudes responded especially to changes in N_2. The character of the transmissions near κ_0 changed rapidly with any small change in ℓ_2 or N_2. Sketches of typical patterns are shown in Fig. 7.

IV. Reflection Cavity, Long Line and Small Obstacle

One might expect a long line effect, similar to the one discussed above, in connection with a reflection cavity (n_2, ℓ_2) when it is separated from a discontinuity n_1 by a long transmission line ℓ_1 as shown in Fig. 8. Here n_2 is quite large while n_1 is
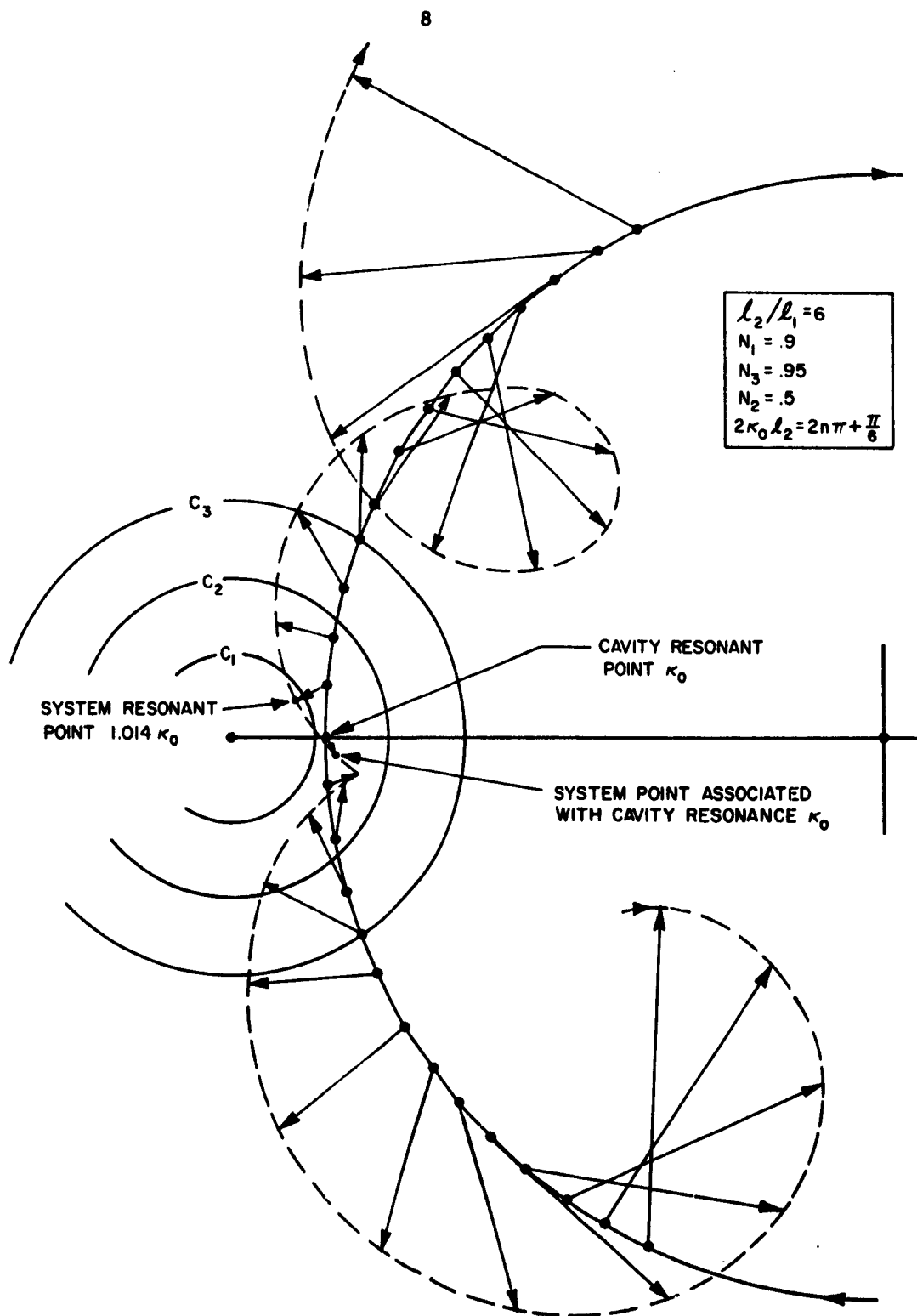
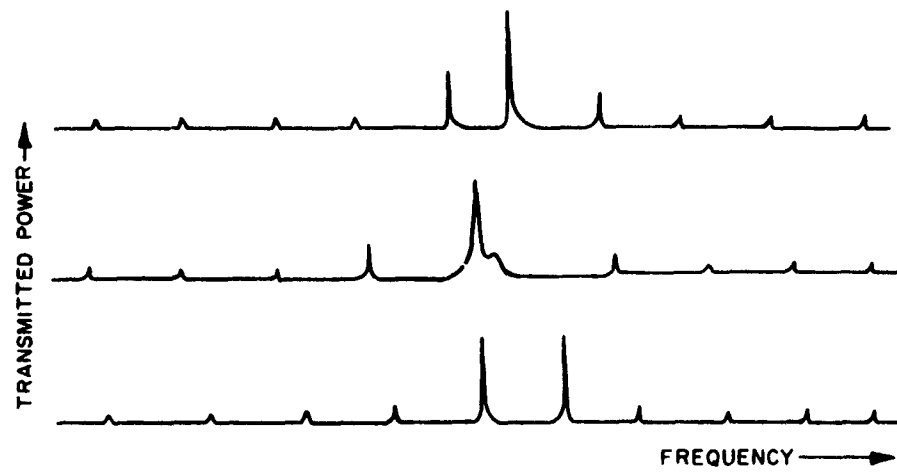Fig. 6 Typical long line effect for a transmission cavity: $\ell_2/\ell_1 = 36$.

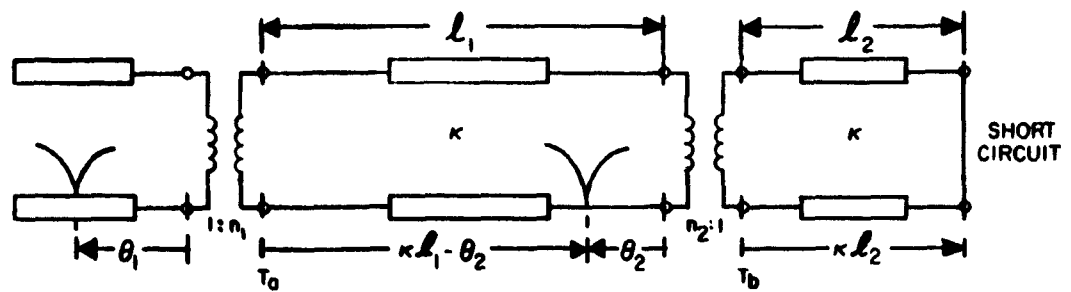Fig. 7 Typical transmission patterns in long line effect.

Fig. 8 Representation of discontinuity, long line and reflection cavity.

much smaller. In view of the lossless idealization of the model, one must seek a criterion of resonance other than the normal absorption which takes place in a real dissipative cavity. Such a criterion can be based on the rate of change with κ of θ, the phase of the input reflection coefficient Γ. While expressions for $d\theta/d\kappa$ are easily derived they do not lead to readily interpretable results unless $d\theta/d\kappa$ is laboriously computed and then plotted against κ. A step by step graphical analysis of the situation, however, based on the "tangent" relation[2] for lossless two-ports does give insight, as will be shown, into the long line effect associated with reflection cavities.

With reference to Fig. 7, the electrical length of the short-circuited transmission line ℓ_2 is $\kappa\ell_2$. As frequency (κ) is varied, it effectively acts as a moving shorting plunger at plane T_b so that, with n_2 assumed constant over the frequency range involved, a nodal shift θ_2 governed by

$$\tan \theta_2 = - n_2^2 \tan(\kappa\ell_2) \tag{9}$$

takes place to the left of transformer $n_2:1$. The frequency dependence of θ_2 is not related to $\kappa\ell_1$; as κ increases θ_2 decreases, so that the associated minimum moves towards the right. The effective short-circuited length of transmission line to the right of plane T_a then is $(\kappa\ell_1 - \theta_2)$ so that the nodal shift θ_1 to the left of transformer $1:n_1$ is given by

$$\tan \theta_1 = \frac{-1}{n_1^2} \tan (\kappa\ell_1 - \theta_2). \tag{10}$$

Note that θ_1 is not θ, the phase of Γ, but is simply related to it by $\theta = 2\theta_1 \pm \pi$.

Fig. 9 shows the various phases plotted qualitatively against $\kappa\ell_2$ in units of π radians. The ordinate has been compressed by a factor of eight, ℓ_1/ℓ_2 has, for convenience, been taken (as small) as seven and $-\theta_1$, rather than θ_1, has been plotted to conserve space and add perspective. The points of maximum (positive) slope of the $-\theta_1$ versus $\kappa\ell_2$ plot indicate the resonances. It can be seen that the cavity resonances which are located at $\kappa\ell_2 = \pi$, 2π are sharp, quite distinct from the "long line" resonances and are influenced by these elements only very slightly. The oscilloscope display of absorptions one would expect to see in this case, but with the normal losses present, is shown below the phase plots. It is understood, of course, that a microwave system could never approach the broad bandedness implied by the example of Fig. 9; nevertheless when ℓ_1/ℓ_2 is substantially larger than the value used here, the phase behavior in a reasonable band near cavity resonance is of the type shown.
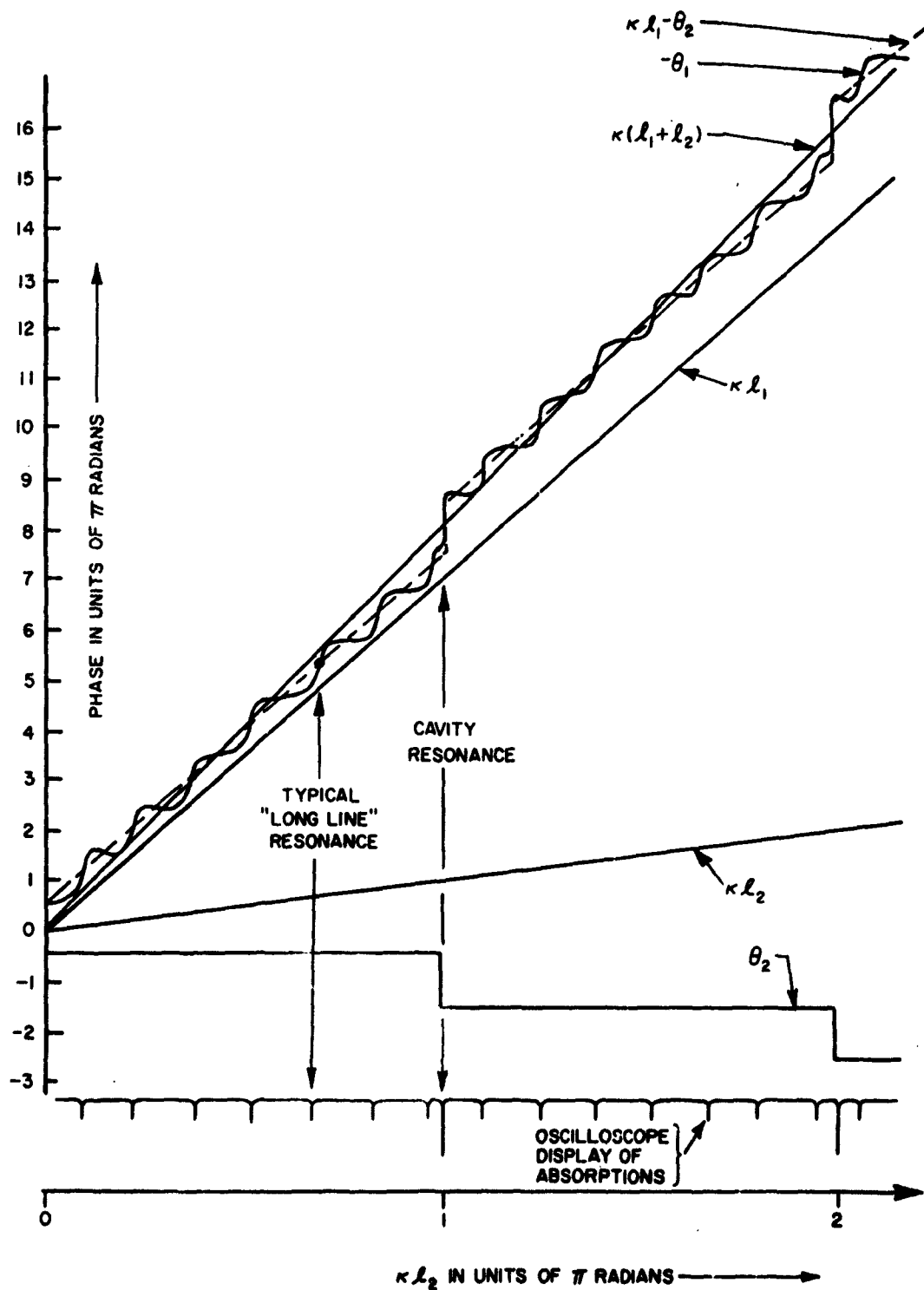
Fig. 9 Typical long line effect for reflection cavity.

The actual resonances over a frequency band of various combinations of reflec-
tion cavities, long lines and obstacles were observed on an oscilloscope in the laboratory.
They were, as expected, distinctly of the two types (cavity and long line) shown in
Fig. 9. Interactions of the two types of resonances in the sense of very small resonant
frequency and absorption changes were apparent only when the resonances were very
close to each other. These effects were much less marked than those observed in
connection with transmission cavities.

V. Conclusions

It has been shown theoretically and verified experimentally that relatively small
obstacles separated from either transmission or reflection cavities can give rise to
many resonances in the neighborhood of the actual cavity resonance itself. In the reflec-
tion cavity case, cavity resonances tend to remain distinct from long line resonances
although a small measure of frequency "pulling" does occur when a long line resonance
is very close to a cavity resonance. Matching out of the power source, directional
couplers, bends, etc., separated from the cavity by a long transmission line, may con-
sequently be indicated when the actual resonant frequ   of the cavity is of importance.
When no long line resonances are located very near the cavity resonant point, however,
such precautions are probably unnecessary in this case.

In the transmission cavity case the long line effect is potentially much more
serious. The actual resonance of the cavity is no longer distinguishable and completely
misleading large resonances (transmissions) of the system may occur. When multiple
resonances are observable near cavity resonances, though one may predominate in am-
plitude, the predominant resonance is in all probability still quite far from the actual
cavity resonant frequency; when no multiple resonances are explicitly observed the
possibility of some frequency pulling, even with shorter "long lines", still exists.
These effects can be especially troublesome when, for example, frequency adjustments
are made by use of a detection system which is subsequently replaced by some other
termination since, presumably, the effect may be present, and in different forms, with
either of these terminations, unless they are well matched. If these effects are to be
avoided, all precautions including the use of well tuned isolators (at the far end from
the cavity of any long lines present) should be seriously considered.

While this long line effect was noted and considered in connection with a cryogenic
system in which long waveguides are required to reach into a dewar, it no doubt occurs
also in a variety of entirely different situations in which cavities and long transmission
lines must be used.

References

1. H. Seidel, "Synthesis of a Class of Microwave Filters," Trans IRE, Vol. MTT-5, April, 1957.

2. N. Marcuvitz, "On the Representation and Measurement of Waveguide Discontinuities," Proc. IRE, Vol. 36, June, 1948.